Universidade de Aveiro
Departamento de Matemática

Cálculo II - Agrupamento 4 **2022/23**

Folha 4: *Soluções*

---

1. (a) Sim; (b) Sim; (c) Não; (d) Sim.

2. (a) $xy' - y = 0$; (b) $y'' = 0$; (c) $xy' - y\ \ln(y) = 0$.

3. $y''' + y' = 0$.

4. (a) $y = C_1 x - \operatorname{sen} x + C_2\,,\quad C_1, C_2 \in \mathbb{R}$.

5. (a) $y = \ln(\operatorname{arctg} x) + C\,,\quad C \in \mathbb{R}$;
   (b) $y = \dfrac{x}{2}\sqrt{1-x^2} + \dfrac{1}{2}\operatorname{arcsen} x + C\,,\quad C \in \mathbb{R}$;
   (c) $y = \dfrac{x^3}{3} + \operatorname{arctg} x + C\,,\quad C \in \mathbb{R}$.

6. (a) $x^2 + y^2 = C\,,\quad C \in \mathbb{R}$;
   (b) $y = C\,x\,,\quad C \in \mathbb{R}$ (compare com o ex. 2(a));
   (c) $\dfrac{x}{t} = C\,\mathrm{e}^{-\frac{1}{x}-\frac{1}{t}}\,,\quad C \in \mathbb{R}$;
   (d) $y = \dfrac{1}{\ln|x^2-1| - C}\,,\quad C \in \mathbb{R}$;

7. (a) $y = \dfrac{1}{x+1}$; (b) $y = -1 + 2e^{2-\sqrt{4+x^2}}$; (c) $y^3 = 4(1+x^3)$.

8. (a) $\ln|y| - \dfrac{x^2}{2y^2} = C,\qquad C \in \mathbb{R}$ ($y = 0$ é solução singular).
   (b) $y = x\,\mathrm{e}^{Ky}\,,\quad x > 0,\qquad K \in \mathbb{R}$.

9. (b) $y = x\,\mathrm{e}^{Cx},\quad x > 0,\qquad C \in \mathbb{R}$.

10. (a) $\operatorname{arctg}\left(\dfrac{y-1}{x-2}\right) - \dfrac{1}{2}\ln\left(1 + \left(\dfrac{y-1}{x-2}\right)^2\right) = \ln|x-2|\,,\quad C \in \mathbb{R}$.
    (b) $(y-x)^2 + 4y = C\,,\quad C \in \mathbb{R}$.

11. (a) $x^2 + x\operatorname{sen} y = C\,,\qquad C \in \mathbb{R}\,$;
    (b) $x^2 + y^2 + 2xe^y - 2yx^2 = C\,,\qquad C \in \mathbb{R}$;
    (c) $y = \dfrac{C - 3x^2}{\ln|x| - 2}\,,\qquad C \in \mathbb{R}$.

12. $x + e^{-x}\operatorname{sen} y = C\,,\quad C \in \mathbb{R}$ (um fator integrante é $\mu(x,y) = e^{-x}\cos y$).

13. (a) $x + y^2 = Cy\,,\quad C \in \mathbb{R}$ (um fator integrante é $\mu(y) = y^{-2}$);
    (b) $yx^2 - \dfrac{x^5}{5} = C\,,\quad C \in \mathbb{R}$ (um fator integrante é $\mu(x) = x,\ \ x > 0$).

14. (a) $y = \dfrac{2}{5}\cos x + \dfrac{1}{5}\operatorname{sen} x + C\,\mathrm{e}^{-2x}\,,\qquad C \in \mathbb{R}$;

(b) $y = -1 + C\,\mathrm{e}^{-\frac{1}{2x^2}}\,, \quad x \neq 0, \quad C \in \mathbb{R};$

(c) $y = (C + x)\sqrt{x^2 + 1}\,, \quad C \in \mathbb{R}.$

15. Comece por verificar que a solução geral possui a forma $y = \dfrac{1}{x} + \dfrac{C}{x^2}\,, \quad C \in \mathbb{R}.$

16. (a) $y = \dfrac{1}{1 + Cx + \ln x}\,, \quad x > 0, \quad C \in \mathbb{R}$ ($y = 0$ é solução singular).

(b) $y^4 = \dfrac{x^2}{C - 4x^5}\,, \quad C \in \mathbb{R}$ ($y = 0$ é solução singular).

17. (a) $y = \dfrac{x^4}{2} + K\,x^2\,, \quad K \in \mathbb{R};$

(b) $y = \dfrac{x}{2}\,\mathrm{cosec}\,x - \dfrac{\cos x}{2} + K\,\mathrm{cosec}\,x\,, \quad K \in \mathbb{R}.$

18. (a) $y = Kx^2 \quad (K \neq 0);$

(b) $y = Ke^x \quad (K \neq 0);$

(c) $x^2 - y^2 = K \quad (K \neq 0).$

19. (a) $y = C_1\,\mathrm{e}^{-x} + \dfrac{\mathrm{sen}\,x}{2} - \dfrac{\cos x}{2};$

(b) $y = C_1\,\mathrm{e}^{x} + C_2\,\mathrm{e}^{-x} + \cos x;$

(c) $y = C_1\,\mathrm{e}^{x} + C_2\,\mathrm{e}^{-2x} + 3x;$

(d) $y = \left(C_1 + C_2 x + \dfrac{x^3}{6}\right)\mathrm{e}^{2x};$

(e) $y = C_1 + (C_2 - x)\,\mathrm{e}^{-x};$

(f) $y = C_1\,\mathrm{sen}\,(2x) + C_2\,\cos(2x) - \dfrac{1}{4}\cos(2x)\,\ln|\sec(2x) + \mathrm{tg}\,(2x)|;$

(g) $y = C_1 + C_2\,\cos x + C_3\,\mathrm{sen}\,x - \dfrac{x}{2}\,\mathrm{sen}\,x;$

(h) $y = C_1\,\mathrm{sen}\,(3x) + C_2\,\cos(3x) + \dfrac{\mathrm{sen}\,x}{8} - \dfrac{\mathrm{e}^{-x}}{10}.$

(C_1, C_2, C_3 são constantes reais arbitrárias).

20. $y = \dfrac{3}{4}(x - \pi)\,\mathrm{e}^{2(\pi - x)} + \dfrac{\mathrm{sen}\,(2x)}{8}.$

21. $y = 1 + \mathrm{e}^{-\mathrm{sen}\,x}\,, \quad x \in \mathbb{R}.$

22. (a) $y = \dfrac{K}{(x^2 + 1)^2}\,, \quad K \in \mathbb{R};$

(b) $y = C_1\,\cos x + C_2\,\mathrm{sen}\,x + x\,\cos x\,, \quad C_1, C_2 \in \mathbb{R};$

(c) $y = C\,\mathrm{e}^{\mathrm{arctg}\,x}\,, \quad C \in \mathbb{R};$

(d) $y = C_1 + C_2\,\cos(2x) + C_3\,\mathrm{sen}\,(2x) + \dfrac{1}{3}\,\mathrm{sen}\,x\,, \quad C_1, C_2 \in \mathbb{R};$

(e) $y = K\,\mathrm{e}^{x^3} - \dfrac{1}{3}\,, \quad K \in \mathbb{R};$

(f) $y = C_1\,\mathrm{e}^{-2x} + (C_2 + C_3 x + 2x^2)\,\mathrm{e}^{x}\,, \quad C_1, C_2, C_3 \in \mathbb{R}.$

23. $y = Cx^2 + x^3 + K\,, \quad C, K \in \mathbb{R}.$

24. (a) –

(b) $y = C_1 x + C_2 e^x\,, \quad C_1, C_2 \in \mathbb{R}.$

(c) $y = C_1 x + C_2 e^x + x^2\,, \quad C_1, C_2 \in \mathbb{R}.$