

Cálculo I - agr. 4 2020/21

2º teste - turma TP4A-6

Resolução

(30 pts) 1. (a) $\int \ln(x^2+1) dx = x \ln(x^2+1) - \int x \frac{2x}{x^2+1} dx$

$$= x \ln(x^2+1) - \int \left(2 - \frac{2}{x^2+1}\right) dx$$

$$= x \ln(x^2+1) - 2x + 2 \operatorname{arctg} x + c, \quad c \in \mathbb{R}$$

$$\text{C.A.: } \frac{2x^2}{x^2+1} = \frac{2x^2+2-2}{x^2+1}$$

$$= 2 - \frac{2}{x^2+1}$$

(30 pts) (b) $\int \frac{x-4}{x^2+x-2} dx$

$$= \int \frac{2}{x+2} - \frac{1}{x-1} dx$$

$$= 2 \ln|x+2| - \ln|x-1| + c,$$

c constante em intervalos.

$$\text{C.A. } x^2+x-2 = (x+2)(x-1)$$

$$\frac{x-4}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \Rightarrow$$

$$x-4 = A(x-1) + B(x+2) \Leftrightarrow$$

$$\Leftrightarrow \underline{x-4} = \underline{Ax} - A + \underline{Bx} + 2B \Leftrightarrow$$

$$\begin{cases} 1 = A+B \\ -4 = -A+2B \end{cases} \Leftrightarrow \begin{cases} A=2 \\ B=-1 \end{cases}$$

(40 pts) (c) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

$$= \int \frac{(2 \sin t)^2}{2 \cos t} \cdot 2 \cos t dt$$

$$= \int 4 \sin^2 t dt$$

$$= \int 2(1 - \cos 2t) dt$$

$$= 2t - \sin 2t \stackrel{+c}{=} 2t - 2 \sin t \cdot \cos t + c$$

$$= 2 \arcsin \frac{x}{2} - x \cdot \frac{\sqrt{4-x^2}}{2} + c, \quad c \in \mathbb{R}.$$

$$\text{C.A. } x = 2 \sin t, \quad t \in]-\pi/2, \pi/2[$$

$$\sqrt{4-x^2} = \sqrt{4-4 \sin^2 t} = \sqrt{4 \cos^2 t}$$

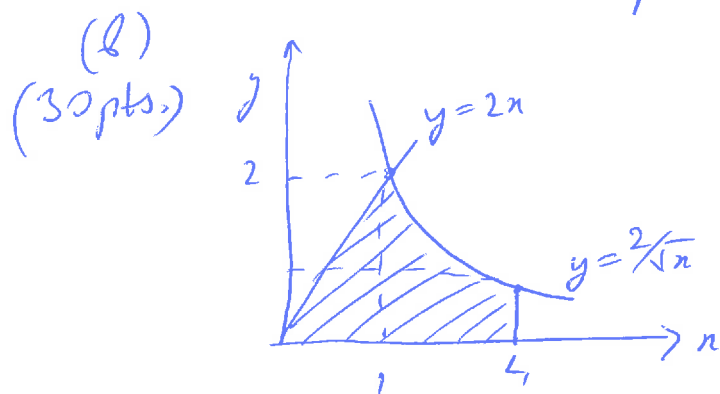
$$= 2|\cos t| = 2 \cos t; \quad dx = 2 \cos t dt$$

$$t = \arcsin \frac{x}{2}$$

$$2. A = \{(x, y) \in \mathbb{R}^2 : y \geq 0, y \leq 2x, y \leq \frac{2}{\sqrt{x}}, 0 \leq x \leq 4\} \quad | \text{2 de 2}$$

(a) (10 pts.) $\begin{cases} y = 2x \\ y = \frac{2}{\sqrt{x}} \end{cases} \Leftrightarrow \begin{cases} x = \frac{1}{\sqrt{x}} \\ x\sqrt{x} = 1 \end{cases} \Leftrightarrow \begin{cases} y = 2 \\ x = 1 \end{cases}$

O ponto de interseção é $(1, 2)$.



(c) (30 pts.)

$$\text{Area}(A) = \int_0^1 2x \, dx + \int_1^4 \frac{2}{\sqrt{x}} \, dx = [x^2]_0^1 + \left[2 \frac{x^{1/2}}{1/2} \right]_1^4$$

$$= 1 + [4\sqrt{x}]_1^4 = 1 + 8 - 4 = 5.$$

3. (a) (10 pts.) $S(f, P_n, C_n) = \sum_{i=1}^n f(\xi_i) \cdot (x_i - x_{i-1})$, sendo

$$P_n = \{a = x_0 < x_1 < \dots < x_{n-1} < x_n = b\}, \quad C_n = \{\xi_1, \dots, \xi_n\}.$$

(b) (20 pts.) 1º Método. Sejam $P_n = \{0 < \frac{1}{n} < \dots < \frac{n-1}{n} < 1\}$, $C_n = \{\frac{1}{n}, \dots, \frac{n-1}{n}, 1\}$; então $S(f, P_n, C_n)$

$$= \sum_{i=1}^n f\left(\frac{i}{n}\right) \cdot \frac{1}{n} = \sum_{i=1}^n \frac{i}{n} \cdot \frac{1}{n} = \sum_{i=1}^n \frac{1}{n^2} \quad \text{e} \quad \lim_{n \rightarrow \infty} S(f, P_n, C_n) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n^2} = \infty.$$

2º Método. Sejam $P_n = \{0 < \frac{1}{n} < \dots < \frac{n-1}{n} < 1\}$ e $C_n = \{\xi_1, \dots, \xi_n\}$, sendo $\xi_1 = \frac{1}{n^2} \in [0, \frac{1}{n}]$. Então $S(f, P_n, C_n) = \sum_{i=1}^n f(\xi_i) \cdot \frac{1}{n}$

$$> f(\xi_1) \cdot \frac{1}{n} = f\left(\frac{1}{n^2}\right) \cdot \frac{1}{n} = n^2 \cdot \frac{1}{n} = n \rightarrow \infty. \quad \text{Em ambos os casos, a sequência de somas de Riemann diverge para o infinito.}$$