

1º teste - turma TP4B-3

Resolução

1.  $f(x) = \arccos(4x - 4x^2)$ .

(a)  $D_f = \{x \in \mathbb{R} : 4x - 4x^2 \in \underbrace{D_{\arccos}}_{[-1, 1]}\}$

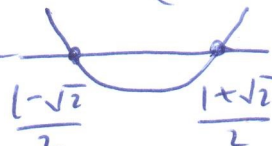
(50 pontos)

C.A. :  $-1 \leq 4x - 4x^2 \leq 1 \Leftrightarrow$

$\Leftrightarrow (4x - 4x^2 - 1 \leq 0 \wedge 4x - 4x^2 + 1 \geq 0)$

$\Leftrightarrow (4x^2 - 4x + 1 \geq 0 \wedge 4x^2 - 4x - 1 \leq 0)$

$\Leftrightarrow ((2x-1)^2 \geq 0 \wedge 4x^2 - 4x - 1 \leq 0) \Leftrightarrow 4x^2 - 4x - 1 \leq 0$

  $4x^2 - 4x - 1 = 0 : x = \frac{4 \pm \sqrt{16 + 16}}{8} = \frac{1 \pm \sqrt{2}}{2}$ .

$\therefore D_f = \left[ \frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2} \right]$ .

(b)  $f'(x) = -\frac{(4x - 4x^2)'}{\sqrt{1 - (4x - 4x^2)^2}} = -\frac{4 - 8x}{\sqrt{1 - (4x - 4x^2)^2}}$  em

$f'(x) \geq 0 \Leftrightarrow 8x - 4 > 0 \Leftrightarrow x > \frac{1}{2}$ .  $\left] \frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2} \right[ \setminus \left\{ \frac{1}{2} \right\}$ .

	$\frac{1-\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{1+\sqrt{2}}{2}$
$f'$	-	+	
$f$	$\searrow$		$\nearrow$

$f$  é contínua em  $\left[ \frac{1-\sqrt{2}}{2}, \frac{1+\sqrt{2}}{2} \right]$ ,

estrit. decrescente em  $\left[ \frac{1-\sqrt{2}}{2}, \frac{1}{2} \right]$

e estrit. crescente em  $\left[ \frac{1}{2}, \frac{1+\sqrt{2}}{2} \right]$ ,

logo existe um mínimo (absoluto) no ponto  $x = \frac{1}{2}$ ,

$f\left(\frac{1}{2}\right) = \arccos(1) = 0$ , e 2 máximas nos pontos

$\frac{1-\sqrt{2}}{2}$  e  $\frac{1+\sqrt{2}}{2}$   $f\left(\frac{1-\sqrt{2}}{2}\right) = \arccos\left(4\frac{1-\sqrt{2}}{2} - 4\left(\frac{1-\sqrt{2}}{2}\right)^2\right)$   
 $= \arccos(-1) = \pi$ ,  $f\left(\frac{1+\sqrt{2}}{2}\right) = \arccos\left(4\frac{1+\sqrt{2}}{2} - 4\left(\frac{1+\sqrt{2}}{2}\right)^2\right) = \arccos(-1) = \pi$ ,

a final nos pontos  $\frac{1-\sqrt{2}}{2}$  e  $\frac{1+\sqrt{2}}{2}$  o valor do máximo é comum, sendo absoluto.