Universidade de Aveiro
Departamento de Matemática

Cálculo I — Agrupamento 4 2019/2020

Soluções dos Exercícios Propostos da Ficha de Exercícios 2

1. (a) $x^3 + \frac{5}{2}x^2 + 7x + c, \quad c \in \mathbb{R}$
   (b) $\frac{3}{4}\sqrt[3]{x^4} + c, \quad c \in \mathbb{R}$
   (c) $\frac{x^7}{7} + \frac{x^4}{2} + x + c, \quad c \in \mathbb{R}$
   (d) $\frac{(\operatorname{arctg} x)^2}{2} + c, \quad c \in \mathbb{R}$
   (e) $\ln|1 + x^3| + c, \quad c \in \mathbb{R}$
   (f) $-\frac{1}{6x^6} + c, \quad c \in \mathbb{R}$
   (g) $\frac{1}{8}\ln(2 + 4x^2) + \frac{\sqrt{2}}{4}\operatorname{arctg}(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
   (h) $\operatorname{sen} x^4 + c, \quad c \in \mathbb{R}$
   (i) $-\sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
   (j) $-\frac{\cos^6 x}{6} + c, \quad c \in \mathbb{R}$
   (k) $-\ln|\cos x| + c, \quad c \in \mathbb{R}$
   (l) $\frac{(\ln x)^2}{2} + c, \quad c \in \mathbb{R}$
   (m) $e^{\operatorname{tg} x} + c, \quad c \in \mathbb{R}$
   (n) $\frac{1}{2\ln 7}7^{x^2} + c, \quad c \in \mathbb{R}$
   (o) $-\frac{\sqrt{2}}{2}\cos(\sqrt{2}x) + c, \quad c \in \mathbb{R}$
   (p) $\frac{x^2}{2} + \ln|x| + c, \quad c \in \mathbb{R}$
   (q) $-\frac{1}{5\sqrt{7+5x^2}} + c, \quad c \in \mathbb{R}$
   (r) $\frac{1}{4}\operatorname{arctg}(x^4) + c, \quad c \in \mathbb{R}$
   (s) $\frac{5}{3}\operatorname{arcsen}(x^3) + c, \quad c \in \mathbb{R}$
   (t) $\frac{\sqrt{7}}{7}\operatorname{arctg}\left(\frac{x}{\sqrt{7}}\right) + c, \quad c \in \mathbb{R}$

2. $F(x) = 2\ln|x| - \frac{3}{x} - 2$

3. $\frac{\pi}{8}(\sqrt{2} - 2)$

4. $F(x) = -\frac{1}{x} + x - \frac{3}{2}$

5. $g(x) = \operatorname{arctg}(\operatorname{arctg} x) - \operatorname{arctg}(\pi/2)$

6. (a) $x\operatorname{sen} x + \cos x + c, \quad c \in \mathbb{R}$
   (b) $x^2\operatorname{sen} x + 2x\cos x - 2\operatorname{sen} x + c, \quad c \in \mathbb{R}$
   (c) $-\frac{2x+3}{3}e^{-3x} - \frac{2}{9}e^{-3x} + c, \quad c \in \mathbb{R}$
   (d) $x(\ln^2 x - 2\ln x + 2) + c, \quad c \in \mathbb{R}$
   (e) $\frac{-e^{2x}\cos x + 2e^{2x}\operatorname{sen} x}{5} + c, \quad c \in \mathbb{R}$
   (f) $\frac{x\operatorname{sen}(\ln x) - x\cos(\ln x)}{2} + c, \quad c \in \mathbb{R}$
   (g) $x\operatorname{arcsen} x + \sqrt{1 - x^2} + c, \quad c \in \mathbb{R}$
   (h) $\frac{x^2}{2}\operatorname{arcsen}(x^2) + \frac{1}{2}\sqrt{1 - x^4} + c, \quad c \in \mathbb{R}$

(i) $x\text{arctg}\, x - \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$

(j) $x\text{arctg}\, \frac{1}{x} + \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$

(k) $\frac{2}{3}\sqrt{x^3}\ln x - \frac{4}{9}\sqrt{x^3} + c, \quad c \in \mathbb{R}$

(l) $\frac{\text{sen}^2 x}{2} + c, \quad c \in \mathbb{R}$

7. (a) $-\ln|\text{cosec}\, x + \text{cotg}\, x| + c, \quad c \in \mathbb{R}$

(b) $\frac{\text{tg}^4 x}{4} + c, \quad c \in \mathbb{R}$

(c) $-\text{cotg}\, x - x + c, \quad c \in \mathbb{R}$

(d) $\frac{1}{2}\theta + \frac{1}{4}\text{sen}\,(2\theta) + c, \quad c \in \mathbb{R}$

(e) $\frac{1}{2}x - \frac{1}{4}\text{sen}\,(2x) + c, \quad c \in \mathbb{R}$

(f) $-\cos t + \frac{1}{3}\cos^3 t + c, \quad c \in \mathbb{R}$

(g) $\frac{\text{tg}^3 x}{3} + \text{tg}\, x - x + c, \quad c \in \mathbb{R}$

(h) $-\frac{1}{3}\cos(3x) + \frac{1}{5}\text{sen}\,(5x) + c, \quad c \in \mathbb{R}$

(i) $\frac{\text{tg}^2 x}{2} + c, \quad c \in \mathbb{R}$

(j) $-\frac{\cos^3 x}{3} + \frac{2}{5}\cos^5 x - \frac{\cos^7 x}{7} + c, \quad c \in \mathbb{R}$

(k) $\frac{1}{16}x - \frac{1}{64}\text{sen}\,(4x) + \frac{1}{48}\text{sen}^3(2x) + c, \quad c \in \mathbb{R}$

(l) $\frac{1}{12}\text{sen}\,(6x) + \frac{1}{8}\text{sen}\,(4x) + c, \quad c \in \mathbb{R}$

(m) $\text{sen}\,(\ln x) + c, \quad c \in \mathbb{R}$

(n) $-\frac{\cos(x^6)}{6} + c, \quad c \in \mathbb{R}$

(o) $-\frac{1}{2}(\arccos x)^2 + \sqrt{1-x^2} + c, \quad c \in \mathbb{R}$

(p) $\frac{1}{2}\text{sen}\,(\ln(x^2)) + c, \quad c \in \mathbb{R}$

8. (a) $\frac{3}{7}\ln|x-1| + \frac{4}{7}|x+6| + c, \quad c \in \mathbb{R}$

(b) $\frac{1}{8}\ln|\,x-1\,| - \frac{1}{8}\ln|\,x+1\,| + \frac{1}{4(x+1)} + \frac{1}{4(x+1)^2} + c, \quad c \in \mathbb{R}$

(c) $\frac{1}{12}\ln|x+2| - \frac{1}{24}\ln(x^2-2x+4) + \frac{\sqrt{3}}{12}\text{arctg}\left(\frac{x-1}{\sqrt{3}}\right) + c, \quad c \in \mathbb{R}$

(d) $\frac{x^3}{3} + 5x + 8\ln|\,\frac{x-3}{x+3}\,| + c, \quad c \in \mathbb{R}$

(e) $-\frac{3}{4}\ln|\,x\,| - \frac{1}{4x} + \frac{13}{16}\ln|\,x-2\,| + \frac{15}{16}\ln|x+2| + c, \quad c \in \mathbb{R}$

(f) $\frac{1}{4}(4x + \ln|x-1| - \ln|x+1| - 2\text{arctg}\, x) + c, \quad c \in \mathbb{R}$

(g) $\ln|x| - \frac{1}{2}\ln(1+x^2) + \frac{1}{2(x^2+1)} + c, \quad c \in \mathbb{R}$

(h) $\frac{1}{2}\ln(x^2+4x+5) - \text{arctg}\,(x+2) + c, \quad c \in \mathbb{R}$

9. (a) $-\frac{2}{3}(1-x)\sqrt{1-x} - \frac{2}{7}(1-x)^3\sqrt{1-x} + \frac{4}{5}(1-x)^2\sqrt{1-x} + c, \quad c \in \mathbb{R}$

(b) $\frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6\text{arctg}\,\sqrt[6]{x} + c, \quad c \in \mathbb{R}$

(c) $\frac{1}{48}(2x+5)^{12} - \frac{5}{44}(2x+5)^{11} + c, \quad c \in \mathbb{R}$

(d) $-\frac{\sqrt{9-x^2}}{9x} + c, \quad c \in \mathbb{R}$

(e) $\arccos\frac{1}{x} + c, \quad c \in \mathbb{R}$

(f) $-\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x} + \frac{2}{x}\right| + c, \quad c \in \mathbb{R}$

(g) $\frac{3\sqrt{2}}{4}\text{arcsen}\left(\sqrt{\frac{2}{3}}\,x\right) + \frac{1}{2}x\sqrt{3-2x^2} + c, \quad c \in \mathbb{R}$

(h) $2\text{arcsen}\,\frac{x+1}{\sqrt{2}} - \frac{(x+1)\sqrt{2-(x+1)^2}}{2} + 2\sqrt{2-(x+1)^2} + c, \quad c \in \mathbb{R}$

(i) $\frac{\sqrt{x^2-7}}{7x} + c, \quad c \in \mathbb{R}$

(j) $\frac{3}{2}\sqrt[3]{2x+3} - 3\sqrt[6]{2x+3} + 3\ln(\sqrt[6]{2x+3}+1) + c, \quad c \in \mathbb{R}$

(k) $2e^{\sqrt{x}}(\sqrt{x}-1) + c, \quad c \in \mathbb{R}$

(l) $\frac{2}{3}\left(\sqrt{1+\ln x}\right)^3 - 2\sqrt{1+\ln x} + c, \quad c \in \mathbb{R}$

10. (a) $-\sqrt{3-x^2} + \operatorname{arcsen}\frac{x}{\sqrt{3}} + c, \ c \in \mathbb{R}$

(b) $\frac{3}{8}x - \frac{1}{4}\operatorname{sen}(2x) + \frac{1}{32}\operatorname{sen}(4x) + c, \quad c \in \mathbb{R}$

(c) $\frac{1}{2}\operatorname{arctg}(\frac{x+1}{2}) + c, \quad c \in \mathbb{R}$

(d) $\ln|\sqrt{\frac{2+x^2}{2}} + \frac{x}{\sqrt{2}}| + c, \quad c \in \mathbb{R}$

(e) $-2\cos\sqrt{x} + c, \quad c \in \mathbb{R}$

(f) $3\ln|x-3| - 2\ln|x-2| + c, \quad c \in \mathbb{R}$

(g) $\operatorname{arcsen}(x-1) + c, \quad c \in \mathbb{R}$

(h) $\frac{(1+x^2)^2\sqrt{1+x^2}}{5} + c, \quad c \in \mathbb{R}$

(i) $x - 2\sqrt{x} + 2\ln(1+\sqrt{x}) + c, \quad c \in \mathbb{R}$

(j) $\frac{x^2}{2}\ln x - \frac{x^2}{4} + c, \quad c \in \mathbb{R}$

(k) $\frac{1}{4}x - \frac{1}{8}\ln(e^{2x}+4) + \frac{1}{2}\operatorname{arctg}\frac{e^x}{2} + c, \quad c \in \mathbb{R}$

(l) $\frac{x^2+1}{2}\operatorname{arctg} x - \frac{1}{2}x + c, \quad c \in \mathbb{R}$

(m) $-\frac{1}{2(1-\cos x)^2} + c, \quad c \in \mathbb{R}$

(n) $(\frac{2}{3}x^3 + 3x)\operatorname{arctg} x - \frac{1}{3}x^2 - \frac{7}{6}\ln(1+x^2) + c, \quad c \in \mathbb{R}$

(o) $\ln\left|\frac{x+1+\sqrt{(x+1)^2-4}}{2}\right| + c, \quad c \in \mathbb{R}$

(p) $2\sqrt{1+e^x} + \ln|\sqrt{1+e^x}-1| - \ln(\sqrt{1+e^x}+1) + c, \quad c \in \mathbb{R}$

(q) $2\operatorname{arctg}\sqrt{e^x-1} + c, \quad c \in \mathbb{R}$

(r) $-2\sqrt{\cos x} + \frac{2}{5}\sqrt{\cos^5 x} + c, \quad c \in \mathbb{R}$

(s) $\frac{1}{2}\ln(\ln^2 x + 1) + c, \quad c \in \mathbb{R}$

(t) $\frac{1}{2}e^{x^2}(x^2-1) + c, \quad c \in \mathbb{R}$

(u) $-\ln|x-2| + \frac{5}{4}\ln|x-3| - \frac{1}{4}\ln|x+1| + c, \quad c \in \mathbb{R}$

(v) $2\sqrt{\operatorname{tg} x - 1} + c, \quad c \in \mathbb{R}$

(w) $-\ln|x| - \frac{1}{2x^2} + \frac{1}{2}\ln(1+x^2) + c, \quad c \in \mathbb{R}$

(x) $\frac{1}{3}(2\ln|x-1| - \ln(x^2+x+1)) + c, \quad c \in \mathbb{R}$

11. $-\frac{3}{2}\frac{1}{4+x^2} + \frac{7}{16}\operatorname{arctg}(\frac{x}{2}) + \frac{7}{8}\frac{x}{4+x^2} + c, \quad c \in \mathbb{R}$

12. (a) $\frac{2}{3}\sqrt{1+x^3} + c, \quad c \in \mathbb{R}$

(b) $-\frac{\sqrt{1+x^2}}{x} + c, \quad c \in \mathbb{R}$

(c) $\frac{1}{2}\left(\ln(x^2+1) - 2\ln|x| + 6\operatorname{arctg} x\right) + c, \quad c \in \mathbb{R}$

(d) $\frac{x}{2} - \frac{1}{4}\ln(e^{2x}+2) + c, \quad c \in \mathbb{R}$

(e) $\frac{1}{1+\operatorname{sen} x} + c, \quad c \in \mathbb{R}$

(f) $\frac{x^2+1}{2}\ln(1+x^2) - \frac{x^2}{2} + c, \quad c \in \mathbb{R}$

(g) $\operatorname{sen} x \cdot \ln(\operatorname{sen} x) - \operatorname{sen} x + c, \quad c \in \mathbb{R}$

13. $f(x) = 2x^3 + 2x + 1$

14. $f(x) = 2\ln(e^x+3) - \ln 4$

15. $f(x) = \ln\left(\frac{x^2-2x+2}{x^2}\right) + 3\operatorname{arctg}(x-1) - \frac{3\pi}{2}$